CSCI 103
More Recursion & Depth First Search

Mark Redekopp
Recursive Flood Fill

• Recall the recursive algorithm for flood fill?
  – Base case: black pixel, out-of-bounds
  – Recursive case: Mark current pixel black and then recurse on your neighbors

```c
void flood_fill(int r, int c)
{
    if(r < 0 || r > 255 )
        return;
    else if ( c < 0 || c > 255){
        return;
    }
    else if(image[r][c] == 0){
        return;
    }
    else {
        // set to black
        image[r][c] = 0;
        flood_fill(r-1,c);  // north
        flood_fill(r,c-1);  // west
        flood_fill(r+1,c);  // south
        flood_fill(r,c+1);  // east
    }
}
```
Recursive Ordering

• Give the recursive ordering of all calls for recursive flood fill assuming N, W, S, E exploration order starting at 4,4
  – From what square will you first explore to the west?
  – From what square will you first explore south?
  – From what square will you first explore east?
  – What is the maximum number of recursive calls that will be alive at any point in time?
Recursive Ordering

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- Notice recursive flood fill goes deep before it goes broad
Recursive Helper Functions

• Sometimes we want to provide a user with a simple interface (arguments, etc.)
• But to implement it recursively we need additional arguments to our function
• In that case, we often let the top-level, simple function call a recursive "helper" function that provides the additional arguments needed to do the work
• [http://cs103.usc.edu/websheets/index.php#sqrt](http://cs103.usc.edu/websheets/index.php#sqrt)
  – Find the square root of, x, without using sqrt function...
  – Pick a number, square it and see if it is equal to x
  – Use a binary search to narrow down the value you pick to square
DEPTH FIRST SEARCH
DFS Algorithm

- BFS finishes all closer vertices before moving to further vertices
- DFS finishes all further vertices before closer vertices
- DFS Approach
  - Mark as started [Gray]
  - For each visited neighbor, visit it and perform DFS on all of their unstarted [White] neighbors
  - Only then, mark as finished [Black]
- DFS is recursive!!
- Coloring/marking system avoids issues with cycles (loops) in the graph

DFS-All (G)
1 for each vertex u
2 u.color = WHITE
3 finish_list = empty_list
4 for each vertex u do
5 if u.color == WHITE then
6 DFS-Visit (G, u, finish_list)
7 return finish_list

DFS-Visit (G, u)
1 u.color = GRAY
2 for each vertex v in Adj(u) do
3 if v.color = WHITE then
4 DFS-Visit (G, v)
5 u.color = BLACK
6 finish_list.append(u)
# Depth First-Search

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Finish_list is just so you can see when we're done with a vertex
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finish_list:

d  e

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finish_list:

a b c d e f g h
BFS vs. DFS Algorithm

• BFS and DFS are more similar than you think
  – Do we use a FIFO/Queue (BFS) or LIFO/Stack (DFS) to store vertices as we find them

BFS-Visit (G, start_node)
1 for each vertex u
2 u.color = WHITE
3 u.pred = nil
4 bfsq = new deque
5 bfsq.push_back(start_node)
6 while bfsq not empty
7 u = bfsq.pop_front()
8 if u.color == WHITE
9 u.color = GRAY
10 foreach vertex v in Adj(u) do
11 bfsq.push_back(v)

DFS-Visit (G, start_node)
1 for each vertex u
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3 u.pred = nil
4 finish_list = new deque
5 finish_list.push_back(start_node)
6 while finish_list not empty
7 u = finish_list.pop_back()
8 if u.color == WHITE
9 u.color = GRAY
10 foreach vertex v in Adj(u) do
11 finish_list.push_back(v)
Memory Benefits of DFS

- Do a BFS and then DFS from the root
  - How many vertices are you keeping track of at any point in time (either in your BFS queue or DFS stack/recursive calls)
  - BFS would yield an entire level
    - Notice at level $h$ you would have $2^{h-1}$ nodes
  - DFS only tracks the depth which is just $h$
Challenge

- Copy your Maze search PA and convert it to use recursive DFS approach
- How do you turn the search into a similar, small problem that can be called over and over (i.e. recursively)
  - Search from a single square
- Base cases:
  - Finish, Wall, already visited, (maybe out of bounds)
- Recursive case:
  - Continue search from your neighbors (much like flood fill)
- Do we still need a BFSQ?
  - No, recursive call stack keeps track of our current path
- Do we still need a predecessor array?
  - Yes, to avoid revisiting locations we've already been
  - But we don't need to know our predecessor to retrace our steps (recursive call stack has the path). We just need to know if we've been there...so maybe change the name from predecessor to 'visited' or 'marked'
Activity

• Take your Maze search PA and convert it to use DFS by...
  – Using a deque or vector for your bfsq (maybe change its name to dfsq)
  – Always remove from the back and put in the back [DFS] rather than removing from the front like in BFS
  – Test using your normal maze1.in

• Now download a new maze
  – wget http://ee.usc.edu/~redekopp/cs103/maze_dfs.in

• Before running do a manual run-through of what your code will do on the maze
  – Will it find the shortest path?
  – Will it find a path?

• Verify by running your new DFS maze search on the maze_dfs.in

```
5 5
...F.
.S#.. 
..#..
..#..
.....
maze_dfs.in
```