Instead of “ones, tens, hundreds, . . .” places, binary has “ones, twos, fours, eights, . . .” places.

<table>
<thead>
<tr>
<th>Base</th>
<th>Digits</th>
<th>#digits</th>
<th>“1000” in this base converted to decimal</th>
<th>“205” in this base converted to decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>decimal</td>
<td>0, 1, 2, . . ., 8, 9</td>
<td>10</td>
<td>10^3 = 1000</td>
<td>2 × 10^2 + 0 × 10^1 + 5 × 10^0 = 205</td>
</tr>
<tr>
<td>binary</td>
<td>0, 1</td>
<td>2</td>
<td>2^3 = 8</td>
<td>n/a</td>
</tr>
<tr>
<td>hexadecimal</td>
<td>0, . . ., 9, A, . . ., F</td>
<td>16</td>
<td>16^3 = 4096</td>
<td>2 × 16^2 + 0 × 16^1 + 5 × 16^0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>= 2 × 256 + 0 + 5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>= 517 dec.</td>
</tr>
<tr>
<td>octal</td>
<td>0, 1, 2, 3, 4, 5, 6, 7</td>
<td>8</td>
<td>8^3 = 512</td>
<td>2 × 8^2 + 0 × 8^1 + 5 × 8^0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>= 2 × 64 + 0 × 8 + 5 × 1 = 133 dec.</td>
</tr>
</tbody>
</table>

1. What is the binary integer 101, represented in decimal? 4 + 1 = 5
2. What is the binary integer 1010, represented in decimal? 8 + 2 = 10. (How is this related to the previous answer?) Twice as much as 101
3. What is the binary integer 10100, represented in decimal? 20. (What is the pattern?) Again twice as much since all ones became twice as valuable
4. What is the binary integer 101001, represented in decimal? 41. Twice as much plus one. (Could you write a program to use this approach?) Yes, and it is useful in LFSR!
5. What is the decimal integer 116, represented in binary? Use either of two common approaches:
   - Work right to left; start by determining the rightmost bit.
   - Work left to right; start by determining how many bits this binary number will have.

Right to left: see “Converting from decimal to base b” on booksite §5.1. 116 is even, so ends in a 0, preceded by representation of 116/2 = 58. 58 even so it ends in a 0, etc. ⇒ 1110100
Left to right: biggest power of 2 that fits (≤ 116) is 64, leaving 116-64 = 52. Biggest power of 2 in this remainder is 32. Keep going with remainders, 116=64+32+16+4 = binary 1110100.

6. What are the hexadecimal numbers C, D, and E, expressed in binary? These are twelve, thirteen, fourteen, which are 1100, 1101, 1110.
7. Express the hexadecimal number C0DE as a sum of 4 terms corresponding to the 4 digits. What is the value of this expression when converted to binary? Note that 16 = 2^4, 16^3 = 2^{12} and ×2 shifts us left by one position. C0DE is 12 × 16^3 + 0 × 16^2 + 13 × 16^1 + 14 × 16^0 = 12 × 2^{12} + 13 × 2^4 + 14 = 1100 0000 0000 0000 + 1101 0000 + 1110 = 1100 0000 1101 1110 (C 0 D E)
8. What is the binary number 100100110, represented in hexadecimal? (Avoid using decimal.) Reverse the previous process. 1 0010 0110 and converting each 4 bits to a hex digit, 126
9. Optional: what is the value of DEE+24 in hexadecimal? (Avoid using decimal.) E12, use long addition working right to left
**Bitwise Operators (In Q10 thru Q14, all numbers are in binary)**

10. What is the binary value of 1010 \(\mid\) 110? 1110

11. What is the binary value of 1010 \& 110? 10

12. What is the binary value of 1010 << 10? 101000

13. What is the binary value of 1010 >> 10? 10

14. What is the binary value of 1010 \(\wedge\) 110? 1100

15. What is the value, expressed in hexadecimal, of C05126 \(\wedge\) CBE245 \(\wedge\) C05126? (What is the trick?) Since the order of inputs to xor doesn’t matter, this equals CBE245 \(\wedge\) C05126 \wedge C05126. Since anything xor’ed with itself is 0, this is CBE245 \(\wedge\) 0 = **CBE245**

**16-bit Two’s-Complement Representations**

16. What is the complement of 0101 0000 1100 1111? 1010 1111 0011 0000

17. Give the **16-bit two’s-complement** binary representation of the decimal integer 116 (Use question 5) **0000 0000 0111 0100**

18. Give the 16-bit two’s-complement binary representation of the decimal integer \(-116\) First complement the bits of +116, then add one, giving **1111 1111 1000 1100**

19. What is the 16-bit two’s-complement **hexadecimal** representation of the decimal integer \(-116\)? Like Q8 (converting each 4 bits to a hex digit) **FF8C**

20. What is the decimal representation of the 16-bit two’s-complement hexadecimal number FFFE? Since the first bit is 1, this number is negative. Call this negative number \(X\). Then the binary representation of the positive number \(\neg X\) is obtained by flipping bits (0000 0000 0000 0001) and adding one (0000 0000 0000 0010). So \(\neg X\) is 2, i.e. \(X\) is \(-2\).

**Optional Challenges**

21. What should the binary numbers 0.1 and 0.01 represent? In decimal these are \(10^{-1}\) and \(10^{-2}\). In binary these are likewise \(2^{-1} = 1/2\) and \(2^{-2} = 1/4\)

22. What are the powers of nine in octal? What are the powers of seventeen in hexadecimal?

23. Booksite exercises 5.1.18, 5.1.23, 5.1.25, Booksite creative exercises 5.1.6, 5.1.29