Circuit Basics

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VOLTAGE AND CURRENT
Current and Voltage

• Charge is measured in units of Coulombs

• Current – Amount of charge flowing through a specific point in a certain time period
  – Measured in Amperes (A) = Coulombs per second
  – Current is usually denoted by the variable, I

• Voltage – Electric potential energy
  – Analogous to mechanical potential energy (i.e. F = mgh)
  – Must measure across two points
  – Measured in Volts (V)
  – Common reference point: Ground (GND) = 0V
    • Often really connected to the ground
Current / Voltage Analogy

Charge = Water

Voltage Source = Water Pressure

\[ V_1 - i + V_2 - V_3 + \]

\[ + V_2 - \]

\[ U_2 \]

\[ U_3 \]
Meet The Components

• Most electronic circuits are modeled with the following components

• Resistor
  – Measures how well a material conducts electrons

• Capacitor & Inductor
  – Measures material's ability to store charge and energy

• Transistor
  – Basic amplification or switching technology
Kirchhoff's Laws

- Common sense rules that govern current and voltage
  - Kirchhoff's Current Law (KCL)
  - Kirchhoff's Voltage Law (KVL)

- Kirchhoff's Current Law (KCL)
  - The current flowing into a location (a.k.a. node) must equal the current flowing out of the location
  - The sum of current at any location must equal 0

\[ i_1 + i_2 = i_3 + i_4 \]

An electronic component (e.g. resistor, transistor, etc.)
Kirchhoff's Current Law

- Reminder: KCL says \textbf{current\_in} = \textbf{current\_out}
- Start by defining a direction for each current
  - It does not matter what direction we choose
  - When we solve for one of the currents we may get a "negative" current
  - "Negative" sign simply means the direction is opposite of our original indication
- In the examples to the right the top two examples the directions chosen are fine but physically in violation of KCL...
- ...but KCL helps us arrive at a consistent result since solving for one of the current values indicates...
  - The magnitude of i1 and i2 are the same
  - They always flow in opposite direction of each other (if one flows in the other flows out or vice versa)
Kirchhoff's Laws

- **Kirchhoff's Voltage Law (KVL)**
  - The sum of voltages around a loop (i.e. walking around and returning to the same point) must equal 0
  - Define "polarity" of voltage and then be consistent as you go around the loop...obviously when you solve you may find a voltage to be negative which means you need to flip/reverse the polarity

**KVL says:**

\[ v_1 + v_2 + v_3 = 0 \]
\[ v_1 + v_2 + v_4 + v_5 = 0 \]
\[ -v_3 + v_4 + v_5 = 0 \]
Practice KCL and KVL

- Use KCL to solve for $i_3$, $i_4$, and $i_6$
  - Node A: $6A = 1A + 2A + i_4$
    - $i_4 = 3A$
  - Node C: $1A + 3A = i_3$
    - $i_3 = 4A$
  - U5 and U6 in series thus $i_6=2A$
  - Check Node B: $3A (i_4) + 2A (i_6) + 4A (i_3) = 6A + 3A$

- Use KVL to solve for $v_3$, $v_4$, $v_5$
  - Loop $\{U3,U7\}$: $-V_3 + -5V = 0$
    - $V_3 = -5V$
  - Loop $\{V5,U6,U4\}$: $-V_5 - 2V + 6V = 0$
    - $V_5 = 4V$
  - Loop $\{U1,U2,U3,U8\}$: $1V + 3V + (-5V) + v_8 = 0$
    - $v_8 = 1V$

**Hint:** Find a node or loop where there is only one unknown and that should cause a domino effect
Resistance and Ohms Law

• Measure of how hard it is for current to flow through the substance

• Resistance = Voltage / Current
  - (How much pressure do you have to put to get a current to flow)

• Measured in Ohms (Ω)

• Ohm's Law
  - \( I = \frac{V}{R} \) or \( V = IR \)
  - \( R \uparrow \Rightarrow I \downarrow \)

Schematic Symbol for a Resistor

http://usc.scout.com/2/926916.html
http://www.zimbio.com/photos/Marquise+Lee/Oregon+v+USC/9qQqBuy838Z
Series & Parallel Resistance

- Series resistors = one after the next with no other divergent path
- Parallel resistors = Spanning the same two points
- Series and parallel resistors can be combined to an equivalent resistor with value given as shown...

Series Connections

Series resistors are connected in a sequence, where each resistor is followed by the next one, with no other divergent paths between them. The effective resistance, $R_{eff}$, is the sum of the individual resistances: $R_{eff} = R_1 + R_2$.

Parallel Connections

Parallel resistors are connected in a way that they share the same two points, allowing the current to flow through any of the resistors. The effective resistance, $R_{eff}$, is given by the reciprocal of the sum of the reciprocals of the individual resistances: $R_{eff} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$. 
Solving Voltage & Current

- Given the circuit to the right, let...
  - $V_s = +5V$, $R_1 = 400$ ohms, $R_2 = 600$ ohms

- Solve for the current through the circuit and voltages across each resistors (i.e. $V_1$ and $V_2$)
  - Since everything is in series, KCL teaches us that the current through each component must be the same, let's call it $i$
    - $i = V_s / (R_1 + R_2) = 5/1000 = 5$ mA
  - This alone lets us compute $V_1$ and $V_2$ since Ohm's law says
    - $V_1 = iR_1$ and $V_2 = iR_2$
    - $V_1 = 2V$ and $V_2 = 3V$
  - Though unneeded, KVL teaches us that
    - $V_s - V_1 - V_2 = 0$ or that $V_s = V_1 + V_2$
Voltage Supply Drawings

- The voltage source in the left diagram (i.e. the circle connected to the "Rest of Circuit") is shown in an alternate representation in the right diagram (i.e. the triangle labeled "Vdd")
- In the left diagram we can easily see a KVL loop available
- There is still a KVL loop available in the right diagram

This diagram is an equivalent to the one above.
Voltage Dividers

• Original Problem
  — Vs = +5V, R1 = 400 ohms, R2 = 600 ohms

• Recall our solution
  — i = Vs / (R1 + R2) = 5/1000 = 5 mA
  — V1 = i*R1 = 2V and V2 = i*R2 = 3V

• When two resistors are in series we can deduce an expression for the voltage across one of them
  — i = Vtot / (R1 + R2)
  — V1 = i*R1 and V2 = i*R2
  — Substituting our expression for i:

\[
V1 = V_{tot} \left(\frac{R1}{R1 + R2}\right) \quad \text{and} \quad V2 = V_{tot} \left(\frac{R1}{R1 + R2}\right)
\]

• The voltage across one of the resistors is proportional to the value of that resistor and the total series resistance

If two resistors Rx and Ry are in series then voltage across Rx is:

\[
Rx = V \cdot Rx / (Rx + Ry)
\]
Solving Voltage & Current

• Reconsidering the circuit to the right with...
  – \( V_s = +5V, R_1 = 400 \text{ ohms}, R_2 = 600 \text{ ohms} \)
• Solve for the current through the circuit and voltages across each resistors (i.e. \( V_1 \) and \( V_2 \))
  – We can use the voltage divider concept to immediately arrive at the value of \( V_2 \)
  – \( V_2 = V_{dd} \left[ \frac{R_2}{R_1+R_2} \right] \)
Solving Voltage & Current

• Consider the circuit on the right...
• What is the relationship between V1 and V3?
  – V1 = V3...Do a KVL loop around R3 to R1
• Can you solve for the voltage V1 (in terms of Vs, R1, R2, R3)?
  – Combine R1 and R3 using parallel resistor relationship
  – R1 and R3 can be combined to $R_{\text{eff}} = \frac{R1R3}{R1+R3}$
  – Now use voltage divider since "$R_{\text{eff}}$" and R2 are in series...
  – V1 = Vs[$ R_{\text{eff}} / (R2 + R_{\text{eff}}) $]

• Can you solve for the voltage V2 (in terms of Vs, R1, R2, R3)?
  – KVL says Vs – V1 – V2 = 0. We know Vs and just solved for V1 so we can plug into: V2 = Vs – V1
A Problem...

• Given the following parameters...
  – Vs=5V, R1=4, R2 = 12, R3 = 2 and R4 = 10 ohms.

• Can we use the voltage divider concept to immediately solve the voltage across R2 or do we need to first do some manipulation? What about R4?

• First, find the total equivalent resistance ($R_{eq}$) seen by Vs and then solve for the voltage across each resistor

First collapse this to a single equivalent resistance, $R_{eq}$
A Problem...

• Given the following parameters...
  - $V_s=5 \text{V}$, $R_1=4$, $R_2 = 12$, $R_3 = 2$ and $R_4 = 10 \text{ ohms}$.

• Solve for the voltage across each resistor
  - $R_{tot} = R_1 + \frac{R_2 \times (R_3 + R_4)}{R_2 + R_3 + R_4}$
    $= 10 \text{ ohms}$
  - $i_1 = \frac{V_s}{R_{tot}} = \frac{5}{10} = 0.5 \text{A}$
  - $V_1 = i_1 \times R_1 = 2 \text{V}$
  - $V_2 = 5 \text{V} - 2 \text{V} = 3 \text{V}$ (using KVL)
  - $V_3 = 3 \times \frac{2}{(2+10)}$ (volt. divider)
    $= 3 \text{V} \times \frac{1}{6}$
    $= 0.5 \text{V}$
  - $V_4 = V_2 \times \frac{R_4}{(R_3 + R_4)}$ (volt. divider)
    $= 3 \text{V} \times \frac{5}{6}$
    $= 2.5 \text{V}$