EE 109 Homework 1

Name: _________________________________________
Due: ____________________
Score: ________

Neatly show your work to get full credit and clearly show your final answer.

1.) [5 pts.] Use KCL to solve for $I_0$.

$$\sum_{i=1}^{n} I_i(node) = 0 \rightarrow -3 - 2 + 7 + 6 + I_0 = 0$$

$$\rightarrow I_0 = -8A$$

Another approach:

$$\sum I_{in}(node) = \sum I_{out}(node) \rightarrow 3 + 2 = 7 + 6 + I_0 \rightarrow I_0 = -8A$$

2.) [8 pts.] Use KVL to solve for $V_1$ and $V_2$.

$$\sum v_i(loop) = 0 \rightarrow$$

1) $-15 + v_1 + 6 - 2 = 0 \rightarrow v_1 = 11^V$

2) $+v_2 + 6 = 0 \rightarrow v_2 = -6^V$

3.) [9 pts.] Solve for the currents $i_1$, $i_2$, $i_3$.

$$\sum_{i=1}^{n} I_i(@ node 1) = 0 \rightarrow$$

$-6 + 5 + i_2 = 0 \rightarrow i_2 = 1A$

$$\sum_{i=1}^{n} I_i(@ node 2) = 0 \rightarrow$$

$+6 - 10 - i_1 = 0 \rightarrow i_1 = -4A$

$$\sum_{i=1}^{n} I_i(@ node 3) = 0 \rightarrow$$

$-5 + i_3 + 10 = 0 \rightarrow i_3 = -5A$
4.) [9 pts.] Solve for the voltages V1, V2, V3

\[ \sum_{i=1}^{n} v_i(\text{loop}) = 0 \rightarrow \]

1) \(-10 + v_1 + 4 = 0 \rightarrow v_1 = 6V\)
2) \(-3 - 10 + v_1 + v_3 + 5 = 0 \rightarrow v_3 = 2V\)
3) \(-v_2 + v_1 + v_3 = 0 \rightarrow v_2 = 8V\)

5.) [9 pts.] Solve for the voltages V1, V2, V3 across the respective resistors.

\[ \sum_{i=1}^{n} v_i(\text{loop}) = 0 \rightarrow \]

1) \(-v_3 + 9 = 0 \rightarrow v_3 = 9V\)
2) \(-v_2 + v_3 + 4 = 0 \rightarrow v_2 = 13V\)
3) \(-v_1 - 12 + v_2 = 0 \rightarrow v_1 = 1V\)

6.) [10 pts.] Reduce the resistor network shown below to a single equivalent resistance. Leave your answer in terms of R1, R2, R4, R5, and R6.

\[ R_{eq} = R_1 + \frac{[R_2 \parallel (R_3 + R_4)] + R_5}{R_2 + R_3 + R_4} + R_5 \]
\[ = R_1 + \frac{R_2 \parallel (R_3 + R_4)}{4.4} + R_5 \]
\[ = 3 + \frac{4 + 2 + 2}{4} + 1 = 6\Omega \]

7.) [10 pts.] Reduce the resistor network shown below to a single equivalent resistance assuming the following resistor values: R1=5\(\Omega\), R2=4\(\Omega\), R3=3\(\Omega\), R4=1\(\Omega\), R5=1\(\Omega\), R6=2\(\Omega\), R7=7\(\Omega\).

Hint: Start by combining R4 and R5 then combine those with R6 and keep going...

\[ R_{eq} = R_1 + [R_2 \parallel (R_3 + (R_6 \parallel (R_4 + R_5))] + R_7 \]
\[ R_{eq} = R_1 + [R_2 \parallel (R_3 + (2 \parallel (2))] + R_7 \]
\[ R_{eq} = R_1 + [R_2 \parallel (3 + (1))] + R_7 \]
\[ R_{eq} = R_1 + [4 \parallel (4)] + R_7 \]
\[ R_{eq} = R_1 + [2] + R_7 = 5 + 2 + 7 = 14\Omega \]
8.) [8 pts.] Find an expression for the current \( i_1 \) if \( R_1 = 4\Omega, R_2 = 3\Omega, R_3 = 6\Omega, R_4 = 2\Omega. \)  
Hint: Combine \( R_2, R_3, R_4 \) into an equivalent resistance which will be in series with \( R_1 \). From here you can use a KVL loop or Ohm's law to solve for \( i_1 \).

\[
\sum_{i=1}^{n} v_i(\text{loop}) = 0 \rightarrow 
-20 + R_1.i_1 + (R_2 \parallel R_3 \parallel R_4).i_1 = 0
\]

When 3 or more resistors are in parallel you can work with 2 at a time.

\[
i_1 = \frac{20}{R_1 + ((R_2 \parallel R_3) \parallel R_4)}
\]

\[
i_1 = \frac{20}{4 + ((3 \parallel 6) \parallel 2)} = \frac{20}{4 + \left(\frac{18}{9}\right) \parallel 2} = \frac{20}{4 + \left(\frac{2 \cdot 2}{2 + 2}\right)} = \frac{20}{4} = 4A
\]

9.) [16 pts.] Use the generalized concept of a voltage divider (review your notes/lecture slides) to find expressions for the voltage \( V_1 \) and also \( V_4 \) in the circuit below. Your expression should be in terms of \( V_s \) and \( R_1-R_4 \).

\[
i(R_1) = i(R_4) = \frac{v(s)}{R_1 + R_2 \parallel R_3 + R_4} = \frac{v(s)}{R_1 + R_2 \parallel R_3 + R_4}
\]

\[
V_1 = R_1.i(R_1) = R_1.\frac{v(s)}{R_1 + R_2 \parallel R_3 + R_4}
\]

\[
V_4 = R_4.i(R_4) = R_4.\frac{v(s)}{R_1 + R_2 \parallel R_3 + R_4}
\]

10.) [6 pts.] Look at the circuit from problem 9. If \( R_4 \) is very large (approaches infinity) what would \( V_4 \) be (approximately).
\[
\lim_{R_4 \to \infty} V_4 = \lim_{R_4 \to \infty} R_4 \cdot \frac{v(s)}{R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3}} = \infty \cdot \frac{v(s)}{(R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3})} = \infty \cdot v(s) = 1 \cdot v(s) = v(s)
\]

11.) [5 pts.] Look at the circuit from problem 9. If R3 is very large (approaches infinity) again solve (approximately) for the voltage V4.

\[
\lim_{R_3 \to \infty} V_4 = \lim_{R_3 \to \infty} R_4 \cdot \frac{v(s)}{R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3}} = R_4 \cdot \frac{v(s)}{R_1 + \frac{R_2 \cdot \infty}{\infty}} = R_4 \cdot \frac{v(s)}{R_1 + \frac{\infty}{\infty}} = R_4 \cdot \frac{v(s)}{R_1 + \frac{R_2 \cdot \infty}{\infty} + R_4}
\]

Note: This is the appropriate voltage divider equation had R3 been removed altogether. Thus as a resistor in parallel gets large, it's as if it's not even there.

12.) [5 pts.] Look at the circuit from problem 9. If R3 is effectively 0Ω (i.e. replaced by a wire), solve (approximately) for the voltage V4.

\[
V_4 = R_4 \cdot \frac{v(s)}{R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3}} \to V_4(R_3 = 0) = R_4 \cdot \frac{v(s)}{R_1 + R_4}
\]

Note: This is the appropriate voltage divider equation had R2 and R3 been removed completely. Thus as a resistor in parallel gets small it's as if neither resistor is present.