2-3-4 Trees

2-3-4 Trees are a further generalization of the 2-3 tree.

- In a 2-3-4 tree, every node has either 1, 2, or 3 values.
- If a node has 1 value and is not a leaf, then it has 2 children, and we call it a 2-node.
- If a node has 2 values and is not a leaf, then it has 3 children, and we call it a 3-node.
- If a node has 3 values and is not a leaf, then it has 4 children, and we call it a 4-node.
- All leaves must be at the same level.
- For all 2-nodes, the binary search tree property holds.
- For all 3-nodes with keys $k_1 < k_2$, and subtrees $T_1, T_2, T_3$, the keys in $T_1$ are $\leq k_1$, the keys in $T_2$ are $\geq k_1$ and $\leq k_2$, and the keys in $T_3$ are $\geq k_2$.
- For all 4-nodes with keys $k_1 < k_2 < k_3$, and subtrees $T_1, T_2, T_3, T_4$, the keys in $T_1$ are $\leq k_1$, the keys in $T_2$ are $\geq k_1$ and $\leq k_2$, the keys in $T_3$ are $\geq k_2$ and $\leq k_3$, and the keys in $T_4$ are $\geq k_3$.

Exercise 1. Starting with an empty tree, perform the following operations in sequence.
Insert: 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17
Delete: 17,13,14,7,2,15,12,10,11
B-Trees

B-Trees are a generalization of both 2-3 Trees and 2-3-4 Trees.

- Every tree has a value \( b_{\text{max}} \), indicating its branching factor.
- Sometimes they have a \( b_{\text{min}} \), indicating its minimum branching factor, where \( \frac{b_{\text{max}}+1}{2} \geq b_{\text{min}} \).
- Every node has between \( b_{\text{min}} - 1 \) and \( b_{\text{max}} - 1 \) values, with the exception of the root which does not have a minimum branching factor.
- If a node is not a leaf, it has one more child than it does values, obeying the generalized binary search tree property.
- All leaves are at the same level.

Red-Black Trees

A Red-Black Tree is exactly a 2-3-4 tree that has been broken up into a binary search tree.

- Each node is either red or black.
- A red node can only have black children.
- For any path from root to leaf, there is an equal number of black nodes.
- The root is black.